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Laboratory experiments and numerical simulations on flow between concentric independently rotating cylinders (the Couette-Taylor system) reveal a primary bifurcation to a new state, ribbons, which are traveling waves in the azimuthal direction but standing waves in the axial direction. Other experiments, conducted on a rigid rapidly rotating annulus, are designed to explore parameter regimes characteristic of planetary scale flows. Eastward jets are found to exhibit Rossby waves for a wide range of control parameters, and these jets (or, more precisely, the potential vorticity gradients in the core of the jets) act as a strong barrier to tracer transport; these observations have important implications for the transport of pollutants in oceans and the atmosphere. The behavior of westward jets is found to be markedly different from that of eastward jets. ~~persistent vortices (like the Great Red Spot of Jupiter) are found~~

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FINAL REPORT

OFFICE OF NAVAL RESEARCH

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NAME OF PRINCIPAL INVESTIGATOR: Harry L. Swinney

NAME OF ORGANIZATION: The University of Texas at Austin

ADDRESS OF ORGANIZATION: Physics Department, Austin, TX 78712

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PAPERS SUBMITTED TO REFEREED JOURNALS (Not yet published)

"Traveling waves in axisymmetric convection: the role of sidewall conductivity," D. Barkley and L. S. Tuckerman, *Physica D*, in press (1989).

PAPERS PUBLISHED IN REFEREED JOURNALS

"Laboratory model of a planetary eastward jet," J. Sommeria, S.D. Meyers, and H.L. Swinney, Nature **337**, 58-61 (1989).

"Transformations of matrices into banded form", L. Tuckerman, J. Computational Physics **84**, 360-376 (1989).

"Divergence-free velocity fields in nonperiodic geometrics," L. Tuckerman, J. Computational Physics **80**, 403-441 (1989).

"Steady-state solving via Stokes preconditioning: recursion relations for elliptic operators," L. S. Tuckerman, to appear in *Proc. of the Eleventh Int'l. Conference on Numerical Methods in Fluid Dynamics*, ed. by D. L. Dwoyer, M. Y. Hussaini, and R. G. Voigt (Springer-Verlag, Berlin, 1989), pp. 573-577.

"Traveling waves in axisymmetric convection," in *New Trends in Nonlinear Dynamics and Pattern Formation: the Geometry of Nonequilibrium*, NATO Advanced Study Institute Series, ed. by P. Coullet and P. Huerre (Plenum, New York, 1989).

"Nonlinear standing waves in Couette-Taylor flow," R. Tagg, S. Edwards, H. L. Swinney, and P. S. Marcus, Phys. Rev. **A 39**, 3734-3737 (1989).

"Primary instabilities and bicriticality in flow between counterrotating cylinders," W.F. Langford, M. Golubitsky, R. Tagg, E. Kostelich, and H.L. Swinney, Phys. Fluids **31**, 776-785 (1988).

"Instabilities and chaos in rotating fluids," in *Nonlinear Evolution and Chaotic Phenomena*, ed. by G. Gallavotti and P. W. Zweifel (Plenum Publishing Co., 1988), pp. 319-326.

"A laboratory simulation of the Great Red Spot of Jupiter," J. Sommeria, S.D. Meyers, and H.L. Swinney, Nature **331**, 689-693 (1988).

"Numerical simulation of Jupiter's Great Red Spot," P.S. Marcus, Nature **331**, 693-696 (1988).

"Global bifurcation to traveling waves in axisymmetric convection," L. S. Tuckerman and D. Barkley, Phys. Rev. Lett. **61**, 408-411 (1988).

"A strange attractor in weakly turbulent Couette-Taylor flow," Anke Brandstater and H.L. Swinney, Phys. Rev. A **35**, 2207-2220 (1987).

"Mass transport in turbulent Couette-Taylor flow," W.Y.Tam and H.L. Swinney, Phys. Rev. A36, 1374-1381 (1987).

PAPERS PUBLISHED IN NON-REFEREED JOURNALS

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TECHNICAL REPORTS PUBLISHED

None

BOOKS (AND SECTIONS THEREOF) SUBMITTED FOR PUBLICATION

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INVITED PRESENTATIONS BY H.L. SWINNEY AT TOPICAL OR SCIENTIFIC/TECHNICAL SOCIETY CONFERENCES

10/2/87 Schlumberger (Austin), Seminar
10/9/-10/87 J. H. Taylor Symposium, Rhodes College, Memphis
10/21-23/87 Dynamic Patterns in Complex Systems, Bahia Mar, Florida
12/14/-15/87 Ed Lorenz Symposium, M.I.T.
2/24/88 Duke University, Physics Department Colloquium
4/13/88 Chaos Review Panel, Jason, Scripps Institute of Oceanography
4/20/88 American Physical Society Annual Meeting, Washington, D. C.
4/21/88 Clarkson University, Physics Colloquium
5/4-5/88 Department of Energy Symposium on Energy Engineering Sciences, Argonne
5/12/88 Rutgers Statistical Mechanics Meeting, Newark, NJ
5/16-20/88 Advances in Fluid Turbulence, Los Alamos, New Mexico
7/26-8/5/88 Enrico Fermi International School of Physics, Nonlinear Topics in Ocean Physics, Varenna, Italy (3 lecture course)
7/29/88 National Institute of Optics, Florence, Italy, Seminar
9/15/88 Texas A&M University, Physics Colloquium
9/21-22/88 Chemical Engineering Colloquium, Rice University, Houston, TX
10/2/87 Schlumberger (Austin), Seminar
10/9/-10/87 J. H. Taylor Symposium, Rhodes College, Memphis
10/21-23/87 Dynamic Patterns in Complex Systems, Bahia Mar, Florida
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7/29/88 National Institute of Optics, Florence, Italy, Seminar
9/15/88 Texas A&M University, Physics Colloquium
9/22/88 Chemical Engineering Colloquium, Rice University, Houston, TX
12/2/88 American Society of Mechanical Engineering Annual Meeting, Chicago
12/19-20/88 Conference on Physics of Fully Developed Turbulence, Princeton

<u>CONTRIBUTED</u> PRESENTATIONS AT TOPICAL OR SCIENTIFIC/TECHNICAL SOCIETY CONFERENCES

11/23-25/87 "Stability of Flow Between Counter-Rotating Cylinders", Randall Tagg, Eric J. Kostelich, and Harry L. Swinney, Annual Meeting of the American Physical Society, Division of Fluid Dynamics

11/23-25/87 "Stability of Flow Between Counter-Rotating Cylinders", Randall Tagg, Eric J. Kostelich, and Harry L. Swinney, Annual Meeting of the American Physical Society, Division of Fluid Dynamics

HONORS / AWARDS / PRIZES (H.L. Swinney)

1989	Distinguished Lecturer, Haverford College
1987	Peyton Nalle Rhodes Lecturer, Rhodes College
1987	Halliburton Distinguished Lecturer, Texas Tech University, Lubbock

OTHER RECENT PROFESSIONAL ACTIVITIES (H. L. Swinney)

Director, The Center for Nonlinear Dynamics, University of Texas, 1985-
Co-organizer, Fluid and Plasma Turbulence Conference, Austin, Dec. 7-11, 1987
Co-organizer, Dynamics Days Texas, Houston, January 5-8, 1988
Co-organizer, Dynamics Days Texas, Houston, January 4-7, 1989
Co-organizer, Workshop on Computation of Nonlinear Flows and Instabilities, Austin, 24-25 March 1989
Principal Organizer, Dynamics Days Texas, Austin, January 3-6, 1990
Member of the Organizing Committee, Complex Systems Summer School, Santa Fe, June 1988, June 1989
Organizer of course entitled *CHAOS* — 7 lectures of 3 hours each held on 7 successive Fridays at The University of Texas Applied Research Laboratory, March 25-May 6, 1988
Organizing Committee, Year of Dynamics (1989-90), Institute of Mathematical Analysis, University of Minnesota
Member, Advisory Board for the Warwick Nonlinear Systems Laboratory, 1986-
Member, Science Board of the Santa Fe Institute, 1987-
Member, External Advisory Board, Center for Interdisciplinary Complex Systems, University of Arizona, 1987-

GRADUATE STUDENTS SUPPORTED UNDER CONTRACT, 1 JUNE 1986 — 30 DECEMBER 1988

Bright Dornblaser
William S. Edwards
Andrew Fraser
Steve Meyers
Michael Schatz

POSTDOCTORALS SUPPORTED UNDER CONTRACT, 1 JUNE 1986 — 30 DECEMBER 1988

Eric Kostelich
Randall Tagg
Laurette Tuckerman
Joel Sommeria

Final Report Discussion: Symmetry Breaking Bifurcations and the Growth of Chaos in a Rotating Fluid

ONR Contract N00014-86-K-0385

86 June 01 — 88 December 31

Harry L. Swinney, Principal Investigator, The University of Texas at Austin

Laboratory experiments and numerical simulations have been conducted on instabilities in flow between concentric cylinders (the Couette-Taylor system) and Rossby waves and vortices in eastward and westward jets. In addition, Rayleigh-Bénard convection near the onset of instability has been investigated in numerical simulations and a bifurcation theory analysis. The principal results are summarized briefly as follows:

Couette-Taylor Flow Instabilities and Turbulence

1. **"Primary instabilities and bicriticality in flow between counter-rotating cylinders," W.F. Langford, M. Golubitsky, R. Tagg, E. Kostelich, and H.L. Swinney, Phys. Fluids 31, 776-785 (1988).**

The stability of the basic flow, *Couette flow*, was examined in a linear stability analysis. The analysis method was not new, but this was the first extensive study of the primary instability for a wide range of control parameters (cylinder rotation rates and ratio of radii). Particular attention was focused on the bicritical curves that separate the transitions from Couette flow to flows with different azimuthal wavenumbers m.

2. **"Nonlinear standing waves in Couette-Taylor flow," R. Tagg, S. Edwards, H. L. Swinney, and P. S. Marcus, Phys. Rev. A39, 3734-3737 (1989).**

Recently Demay and Iooss conducted a *nonlinear* stability analysis of Couette flow and discovered a new state — *ribbons*, which are traveling waves in the azimuthal direction but standing waves in the axial direction. This was the first new primary bifurcation predicted for this system since G.I. Taylor's discovery of axisymmetric (Taylor) vortices and spirals in 1923. We have observed the ribbons in laboratory experiments and corroborated our observations with direct numerical simulations at the parameter values where the ribbons were observed.

3. **"Strange attractors in weakly turbulent Couette-Taylor flow," A. Brandstater and H.L. Swinney, Phys. Rev. A 35, 2207-2220 (1987).**

The transition from quasi-periodic to chaotic Couette-Taylor flow was studied by laser doppler velocimetry and flow visualization. Velocity data were used to construct attractors and Poincaré sections. The attractor dimension was found to increase from 2 at the onset of chaos to about 4 at a Reynolds number 50% above the onset of chaos.

4. **"Mass transport in turbulent Couette-Taylor flow," W.Y. Tam and H.L. Swinney, Phys. Rev. A. 36, 1374-1381 (1987).**

Mass transport was studied at very high Reynolds numbers R (up to 10^5) by injecting a pulse of dye into the fluid and and then measuring concentration as a function of time and axial position. Transport in the axial direction was found to be modeled very well by a one-dimensional diffusion process, with the diffusion coefficient D given by a power law, $D \propto R^{\beta}$, where β is in the range $0.69 < \beta < 0.86$, depending on the radius ratio.

5. **"Instabilities and chaos in rotating fluids," H. L. Swinney, in *Nonlinear Evolution and Chaotic Phenomena*, ed. by G. Gallavotti and P. W. Zweifel (Plenum Publishing Co., 1988), pp. 319-326.**

This brief report, contained in a NATO Advanced Study Institute volume, summarizes our laboratory's recent work on Couette-Taylor and geostrophic flows.

Planetary-type Flows

6. "A laboratory simulation of the Great Red Spot of Jupiter," J. Sommeria, S.D. Meyers, and H.L. Swinney, Nature 331, 689-693 (1988).

This experiment demonstrated that a robust large coherent vortex can form spontaneously in a rapidly rotating turbulent shear flow. The fluid was contained in a rigid rotating annulus; a strong westward jet was generated by the action of the Coriolis force on fluid pumped radially outward. The vortices that form in this flow exhibit many of the properties of jovian spots: they have the same sign of vorticity as the background flow, they move along with the surrounding fluid, the width (longitude) to height (latitude) ratio is about equal to two for the large spots, and, most importantly, nearby spots are attracted to one another and merge. Thus this experiment sheds light on a mystery that has endured for more than three centuries, the origin of the Great Red Spot.

7. "Numerical simulation of Jupiter's Great Red Spot," P. S. Marcus, Nature 331, 693-696 (1988).

The equations governing the motion of an inviscid fluid in a model of the jovian atmosphere are solved for a variety of initial conditions. Large vortices are found to form spontaneously in chaotic azimuthal flows. The spots are stable if the vorticity of the spots has the same sign as the shear of the surrounding azimuthal flow. These simulated spots exhibit many of the properties of the jovian spots and of the spots observed in a laboratory experiment (paper number 6), and the numerical simulation yields a new prediction for the vertical structure of the Great Red Spot.

8. "Laboratory model of a planetary eastward jet," J. Sommeria, S.D. Meyers, and H.L. Swinney, Nature 337, 58-61 (1989)

A strong eastward jet is generated in the rotating annulus (used in paper number 6) by reversing the direction of the radial flow. For a wide range of parameters Rossby waves form on a narrow, essentially laminar, eastward jet. The wave velocity and wavelength are accounted for by simple scaling arguments. Experiments with dye injected into the annulus far from the jet show that the dye is rapidly mixed away from the core of the jet, but the jet acts as a strong barrier to tracer transport; photographs obtained hundreds of rotations after the injection dramatically illustrate the sharp barrier formed by the jet core. This could have important implications for the understanding of transport of pollutants in the atmosphere and oceans.

Convection

9. "Global bifurcation to traveling waves in axisymmetric convection," L. Tuckerman and D. Barkley, Phys. Rev. Lett. 61, 408-411 (1988).

We have discovered a traveling wave state in axisymmetric convection in which rolls are continually created at the sidewalls and annihilated at the center. Heuristically, for an aspect ratio of five, the system alternates between four and five rolls in an irreconcilable wavelength selection conflict. We have demonstrated that for conducting side walls this oscillatory state is a heteroclinic orbit: a saddle-node bifurcation that leaves in its wake a limit cycle whose period is infinite at onset. In contrast, for insulating sidewalls, when the central roll disappears, again via saddle-node bifurcation, transition occurs instead to a steady four-roll state. We have interpreted the Prandtl number dependence of the transition threshold in light of a proposed wavelength selection criterion [Y. Pomeau and P. Manneville, *J. Phys*, (Paris) **42**, 1067 (1980)] prohibiting net radial flow in axisymmetric convection.

10. **"Traveling waves in axisymmetric convection: the role of sidewall conductivity," D. Barkley and L. Tuckerman, *Physica D*, in press (1989); see also the paper** in *New Trends in Nonlinear Dynamics and Pattern Formation: the Geometry of Nonequilibrium*, **NATO Advanced Study Institute Series, ed. by P. Coullet and P. Huerre (Plenum, New York, 1989).**

Traveling waves in axisymmetric convection such as we have computed numerically have been predicted by theory to exist, but have never been observed experimentally. We have explained this by showing that the four-roll branch, which intercepts the traveling wave solution, is an isola whose range of existence increases as the ratio of sidewall to fluid conductivity is decreased. Traveling waves will be triggered to occur only if the conductivity of the sidewalls is over 60 times that of the fluid.

Applied Mathematics

11.**"Divergence-free velocity fields in nonperiodic geometries," L.Tuckerman, J. Computational Physics 80, 403-441 (1989).**

One of the recently favored methods of imposing incompressibility in spectral calculations has been the influence matrix method, originally presented [L. Kleiser and U. Schumann, in *Proceedings of the Third GAMM Conference on Numerical Methods in Fluid Mechanics*, ed. by E. H. Hirschel (Vieweg and Sohn, Braunschweig, 1980)] as algebraic equations relating coefficients in one nonperiodic direction. We have developed a matrix formalism for a general geometry, in which the influence matrix method can be recast as an application of the classic Sherman-Morrison-Woodbury formula of numerical linear algebra. This formalism has permitted us to apply the method to cylindrical coordinates, complicated by boundaries in two directions and by the coordinate singularity on the axis.

12. **"Transformations of matrices into banded form," L. Tuckerman, J. Computational Physics, 84, 360-376 (1989).**

Implicit time-stepping in spectral methods required the inversion of elliptic operators acting on orthogonal function expansions. This operation is generally quadratic in the number of basis functions in a nonperiodic (bounded) direction, but can be made linear in at most one Cartesian direction by use of a well-known recursion relation [D. Gottlieb and S. A. Orszag, *Numerical Analysis of Spectral Methods: Theory and Applications* (SIAM Press, Philadelphia,1977)]. We have isolated the relevant property responsible for the existence of such recursion relations, and applied it to spherical and cylindrical geometries.

13. **"Steady-state solving via Stokes preconditioning: recursion relations for elliptic operators," L. Tuckerman, in *Proc. of the Eleventh Int'l. Conf. on Numerical Methods in Fluid Dynamics*,, ed. by D.L.Dwoyer, M.Y. Hussani, and R.G.Voight (Springer-Verlag, Berlin 1989), pp. 573-577.**

Steady-state continuation via Newton's method is an attractive alternative to time stepping, both because dynamical timescales need not be resolved, and because unstable steady states are accessible. However, Newton's method requires the solution of large linear systems involving Jacobian matrices which are full, nonsymmetric, and poorly conditioned. We describe a way of modifying a time-dependent code to perform Newton iteration, in which implicit time stepping automatically acts as an effective preconditioner for the Jacobian matrices. We have been able to compute steady states in 1/30th the time required by a Courant-condition-limited time-stepping code by using the NSPCG biconjugate gradient squared algorithm [T. C. Oppe, W. D. Joubert, and D. R. Kincaid (1988), Publ. 3216 of the Center for Numerical Analysis, University of Texas at Austin].